Aspects of the Design of Low Noise, Negative Resistance, Reflection Mode Transistor Amplifiers

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Abstract —**In conventional microwave transistor amplifiers the transistor is used in transmission mode. This paper considers the use of microwave transistors in negative resistance reflection mode and presents the conditions for optimum noise performance. Possible advantages include the possibility of higher gain in the mm-wave region, which can be achieved by absorbing the parasitic common lead inductance into the feedback circuit designed to generate the negative resistance [1], and the existence of a failsafe mode of operation, in that the failure of the active device or its power supply is likely to lead to a low return loss, resulting in a small insertion loss through the amplifier, which may permit continued although degraded system operation. The latter potential advantage has proved to be of interest to radar system designers.**

I. Introduction

THE principle of negative resistance reflection mode microwave amplification is well known. A considerable body of literature exists on the design of such amplifiers using tunnel diodes, avalanche transit time diodes, masers and parametric devices. The possibility of using GaAs FET's in negative resistance amplifiers, and the potential benefit of higher gain at high frequencies, have been recognized and demonstrated by others [1], [2], but the conditions for optimum noise operation have not been published.

A schematic diagram of a low noise reflection mode amplifier using a generalized two port device is given in Fig. 1.

The feedback and the terminating impedance on port two are chosen to give negative resistance and optimum noise measure at port one. The transforming network is designed to give the required gain and bandwidth characteristics and the circulator is used to separate the input and output signals.

II. Theory

The active device in a reflection amplifier stage is designed to have a reflection coefficient greater than unity. This gives rise to the amplifier gain. In the case of tunnel diode, Gunn diode and avalanche transit time devices, this is caused by junction or bulk effects in the semiconductor material causing negative differential resistance. In the case of parametric amplifiers it is caused by the periodic variation of the diode capacitance by a higher frequency pump.

In a two port device such as a GaAs FET or HEMT an effective negative resistance can be generated on port one by adding feedback, if needed to make the device potentially unstable, and applying a suitable terminating impedance to port two. Methods for achieving negative resistance in this way are well documented in the oscillator literature.

Penfield [3] has shown that the noise measure of a negative resistance reflection amplifier is independent of the three port network used to couple power in and out of the negative resistance device, assuming that the network is lossless.

The noise measure is given by

$$M = -\frac{P_e}{kT_oB} \tag{1}$$

where P_e is the exchangeable noise power of the negative resistance device,

- k is Boltzmann's constant,
- T_o is the reference temperature, by convention 290° K and
- B is the noise measurement bandwidth.

The gain of a negative resistance reflection amplifier, for a given negative resistance device, is determined by the impedance presented to the device by the input transforming network. Penfield's result (1) therefore implies that the noise measure is independent of the amplifier gain.

The task of designing an optimum low noise negative resistance reflection amplifier using a transistor may thus be divided into two parts; firstly choosing the feedback and load port impedances to create a negative resistance one port device with minimum noise measure and secondly designing the input transforming network to give the required gain-frequency response.

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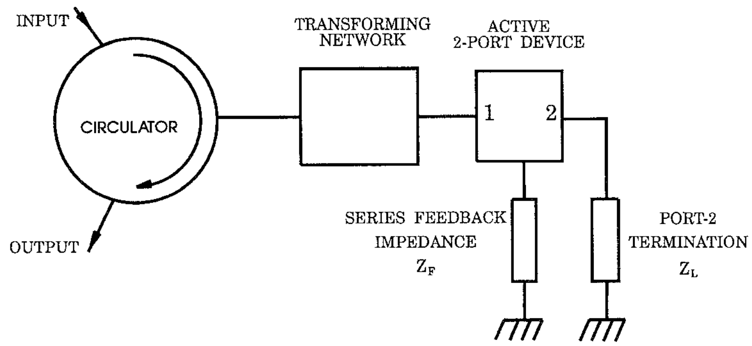

Fig. 1. Schematic diagram of an active two port device configured as a negative resistance amplifier.

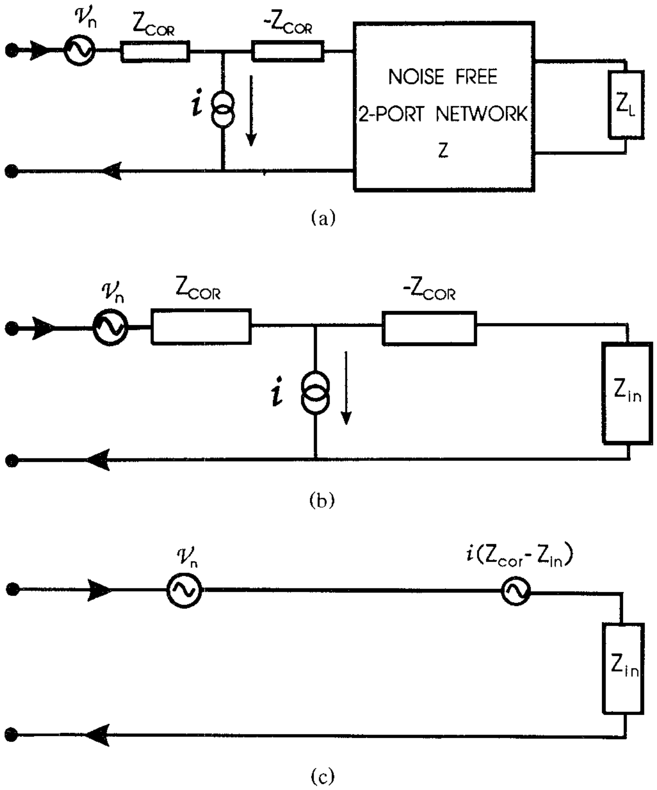

Fig. 2. Noise equivalent circuit for a 2-port device configured as a negative resistance amplifier.

A. Minimum Noise Measure Design

The minimum noise measure design can be derived by considering first the internal noise sources of the transistor as in the treatment of Rothe and Dahlke [4]. Their general model for a noisy two port device is shown in Fig. 2(a) with the output port of the network terminated in a noiseless impedance Z_L. Z_{cor} is the correlation impedance as defined in [4], and v_n and i are uncorrelated noise voltage and current sources.

The input impedance Z_{in} of the two port is given by

$$Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_L + Z_{22}}. \tag{2}$$

The equivalent circuit can be simplified further as shown in Fig. 2(b) and (c). In Fig. 2(c) the two port device with two noise sources has been reduced to a one port device with a single noise voltage source. The exchangeable noise power can be written as

$$P_e = \frac{\overline{|v_{nt}|^2}}{4R_{in}} \tag{3}$$

where $v_{nt} = v_n + i(Z_{cor} - Z_{in})$ is the total instantaneous noise voltage of the one port device, $R_{in} = \mathrm{Re}(Z_{in})$ is the input resistance, which must be negative for a reflection amplifier, and the overbar indicates a time average.

Now by definition v_n and i are totally uncorrelated, so

$$\overline{|v_{nt}|^2} = \overline{|v_n|^2} + \overline{|i|^2} \cdot |Z_{cor} - Z_{in}|^2 \tag{4}$$

$$= 4kT_oB\left(r_n + g_n|Z_{cor} - Z_{in}|^2\right). \tag{5}$$

Substituting (5) into (3) and the result into (1) we obtain the noise measure as a function of Z_{in}:

$$M = -\frac{r_n}{R_{in}} - \frac{g_n}{R_{in}} \cdot |Z_{cor} - Z_{in}|^2. \tag{6}$$

By differentiating (6) with respect to the real and imaginary parts of Z_{in} it can be seen that a minimum value for M is achieved for

$$Z_{in} = -\sqrt{\frac{r_n}{g_n} + R_{cor}^2} + jX_{cor} \tag{7}$$

$$= -Z_{opt} \tag{8}$$

where Z_{opt} is the source impedance for optimum noise figure in conventional transmission mode, as derived by Rothe and Dahlke [4].

The minimum noise measure achieved when $Z_{in} = -Z_{opt}$ is found by substitution into (6):

$$M_{min} = \frac{r_n}{R_{opt}} + \frac{g_n}{R_{opt}}|R_{opt} + R_{cor}|^2$$

$$= 2g_n(R_{opt} + R_{cor})$$

$$= F_{min} - 1 \tag{9}$$

where R_{opt} and R_{cor} are the real parts of Z_{opt} and Z_{cor} respectively and F_{min} is the optimum noise figure in conventional transmission mode, as derived by Rothe and Dahlke [4]. This result is in apparent contradiction of the conclusion of Haus and Adler [10], that the optimum

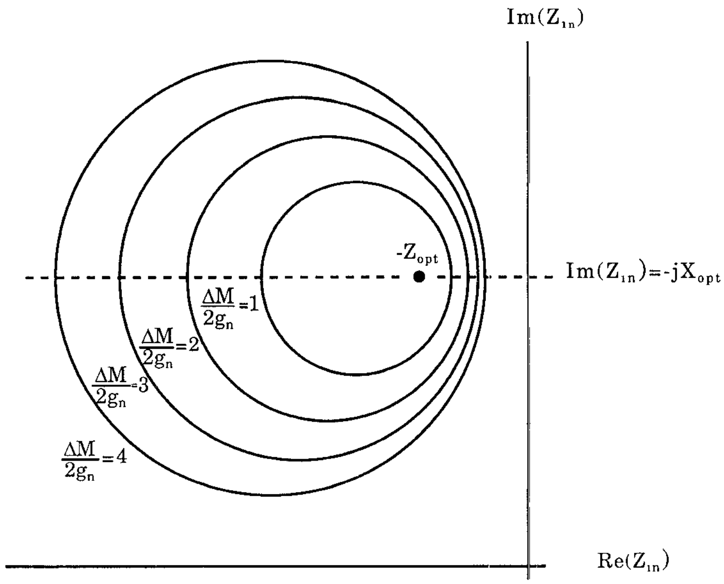

Fig. 3. Circles of constant noise measure in the Z_{in} plane.

noise measure is an invariant property of an active device whilst optimum noise figure is not. The apparent contradiction is resolved by observing that in practice the requirement for the terminating impedance to be noiseless restricts the choice of Z_L to a pure reactance. This constrains the range of values of Z_{in} so that the condition for optimum noise measure in (8) is not generally achievable.

The lowest achievable noise measure subject to this constraint can be determined by studying contours of constant noise measure in the Z_{in} plane, along with the locus of Z_{in} as Z_L takes all imaginary values from $+j\infty$ to $-j\infty$.

From (6) it can be shown that the contour of constant noise measure M_i in the Z_{in} plane is a circle with center C_i and radius R_i where

$$C_i = -\left[R_{opt} + \frac{\Delta M_i}{2g_n}\right] - jX_{opt}$$

$$= -\alpha_i - jX_{opt} \tag{10}$$

$$R_i^2 = \left[R_{opt} + \frac{\Delta M_i}{2g_n}\right]^2 - R_{opt}^2$$

$$= \alpha_i^2 - R_{opt}^2 \tag{11}$$

where

$$\Delta M_i = M_i - M_{min} \tag{12}$$

and

$$\alpha_i = R_{opt} + \frac{\Delta M_i}{2g_n}. \tag{13}$$

The family of circles represented by (10) and (11) is illustrated in Fig. 3. As M_i increases the center of the circle moves to the left and the radius increases. As $M_i \to \infty$, $\text{Re}[C_i + R_i] \to 0$, i.e., the circular contour tends to a straight line on the vertical axis.

The locus of Z_{in} for all imaginary Z_L values is determined from (2), and can be shown to be a circle with center and radius C_o and R_o where

$$C_o = Z_{11} - \frac{Z_{11}Z_{21}}{2\text{Re}(Z_{22})} \tag{14}$$

$$R_o = |K| = \left|\frac{Z_{11}Z_{22}}{2\text{Re}(Z_{22})}\right| \tag{15}$$

The equation of the circular locus in the Z_{in} plane is

$$Z_{in} = C_o + K\tau \tag{16}$$

where

$$\tau = \frac{j(X_L + X_{22}) - R_{22}}{j(X_L + X_{22}) + R_{22}} \tag{17}$$

$X_L = \text{Im}(Z_L)$: $R_{22} = \text{RE}(Z_{22})$ and $X_{22} = \text{Im}(Z_{22})$.

It can be seen that $|\tau| = 1$, for all values of X_L.

If $\text{Re}(C_o - R_o) > 0$ then there is no value of X_L for which $\text{Re}(Z_{in}) < 0$, so that the two port is unconditionally stable and cannot be used for a negative resistance amplifier.

The minimum achievable noise measure with a pure reactive termination on port two is that for which the contour touches the locus of Z_{in}, as illustrated in Fig. 4.

The condition for this minimum noise measure value is given by

$$|C_i - C_o| = R_o + PR_i \tag{18}$$

where

$$P = 1 \text{ if } |Z_{opt} + C_o| > R_o$$

and

$$P = -1 \text{ if } |Z_{opt} + C_o| < R_o.$$

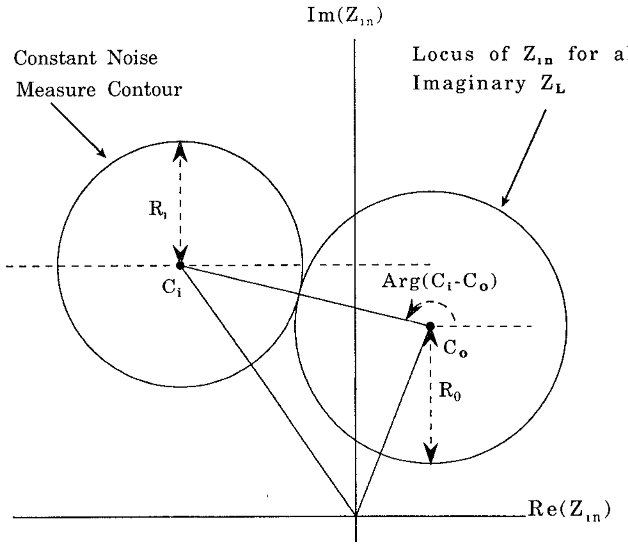

Fig. 4. Graphical solution for optimum noise measure.

Squaring (18) and expanding gives

$$R_o^2+2PR_iR_o+R_i^2=|C_i|^2+|C_o|^2-2\operatorname{Re}(C_i)\operatorname{Re}(C_o)-2\operatorname{Im}(C_i)\operatorname{Im}(C_o). \quad (19)$$

Substituting for C_i and R_i into (19) from (10) and (11) gives

$$2PR_0\sqrt{\alpha_i^2-R_{\mathrm{opt}}^2}=2\operatorname{Re}(C_0)\alpha_i-B \quad (20)$$

where

$$B=R_o^2-|Z_{\mathrm{opt}}+C_o|^2+2R_{\mathrm{opt}}\operatorname{Re}(C_o).$$

Squaring (20) and solving the resulting quadratic equation for α_i gives

$$\alpha_i=\frac{-B\operatorname{Re}(C_o)\pm R_o\sqrt{B^2+4R_{\mathrm{opt}}^2\left(R_o^2-[\operatorname{Re}(C_o)]^2\right)}}{2\left(R_o^2-[\operatorname{Re}(C_o)]^2\right)}. \quad (21)$$

The sign of the second term in the numerator of (21) is chosen such that $\alpha_i>R_{\mathrm{opt}}$, in order to make ΔM_i in (13) positive. The pole in (21) as $\operatorname{Re}(C_o)\to R_o$ corresponds to the approach to unconditional stability.

Hence the optimum reflection mode noise measure for a given two port network with a purely reactive termination on port two is given by

$$M_{\mathrm{opt}}=M_{\min}+2g_n(\alpha_i-R_{\mathrm{opt}})$$
$$=F_{\min}-1+2g_n(\alpha_i-R_{\mathrm{opt}}). \quad (22)$$

The load reactance required to achieve the optimum condition in (22) can be determined by setting

$\arg(K\tau_{\mathrm{opt}})=\arg(C_i-C_o)$, as shown in Fig. 4.

Thus, $\arg(\tau_{\mathrm{opt}})=\arg(C_i-C_o)-\arg(K)=\theta_{\mathrm{opt}}$. (23)

Using (23) and (17), we obtain

$$X_{L\,\mathrm{opt}}=\frac{R_{22}\sin\theta_{\mathrm{opt}}}{1-\cos\theta_{\mathrm{opt}}}-X_{22}. \quad (24)$$

Thus for a given two port network the optimum reflection mode noise measure at port 1 and the load reactance needed on port 2 can be determined. However for an unconditionally stable two port it is necessary to modify the two port by adding external feedback in order to make negative resistance amplification possible.

Even if the two port is potentially unstable without external feedback, the addition of feedback generally enables a lower noise measure to be achieved. The type of feedback considered here is series reactive feedback. Series feedback is used because this is easily incorporated into the common terminal of the transistor and reactive elements are used because these do not contribute additional noise.

To find the optimum series reactive feedback for minimum noise measure, account must be taken of the dependence of the Z-parameters and the noise parameters on the feedback reactance. For the Z-parameters this is straightforward:

$$Z'_{ij}=Z_{ij}+jX_f \quad (25)$$

where X_f is the feedback reactance.

The transformation of two port noise parameters for general network changes has been addressed by Hartmann and Strutt [5]. For the specific case of series reactive feedback the transformation equations become

$$g'_n=\left|\frac{Z_{21}}{Z_{21}+jX_f}\right|^2 g_n \quad (26)$$

$$r'_n=r_n \quad (27)$$

$$Z'_{\mathrm{cor}}=Z_{\mathrm{cor}}+jX_f+\frac{jX_f(Z_{\mathrm{cor}}-Z_{11})}{Z_{21}}. \quad (28)$$

From these transformation equations it is possible to derive the transformations of the familiar two port noise parameters:

$$F'_{\min}=1+2\left(g'_nR'_{\mathrm{cor}}+\sqrt{g'_nr_n+g_n'^2R_{\mathrm{cor}}'^2}\right) \quad (29)$$

$$Z'_{\mathrm{opt}}=\sqrt{\frac{r_n}{g'_n}+R_{\mathrm{cor}}'^2}-jX'_{\mathrm{cor}}. \quad (30)$$

By substituting the transformation equations (25)–(30) into (22) and (24), it is possible to determine the optimum achievable reflection mode noise measure and the associated load reactance as functions of feedback reactance. By plotting the optimum noise measure against feedback reactance the overall optimum can be identified.

A simple program to generate such plots has been written and some example results, covering the NE710-83 device at 2 GHz and 10 GHz, are shown in Fig. 5. The plots show $10\log_{10}(M_{\mathrm{opt}}+1)$ as a function of feedback reactance X_f. This corresponds to the noise figure of the amplifier in dB when matched for high gain. For comparison with the transmission amplifier case the plots also show $10\log_{10}F_{\min}$ as a function of X_f. The required port 2 load reactance is also shown in the plots.

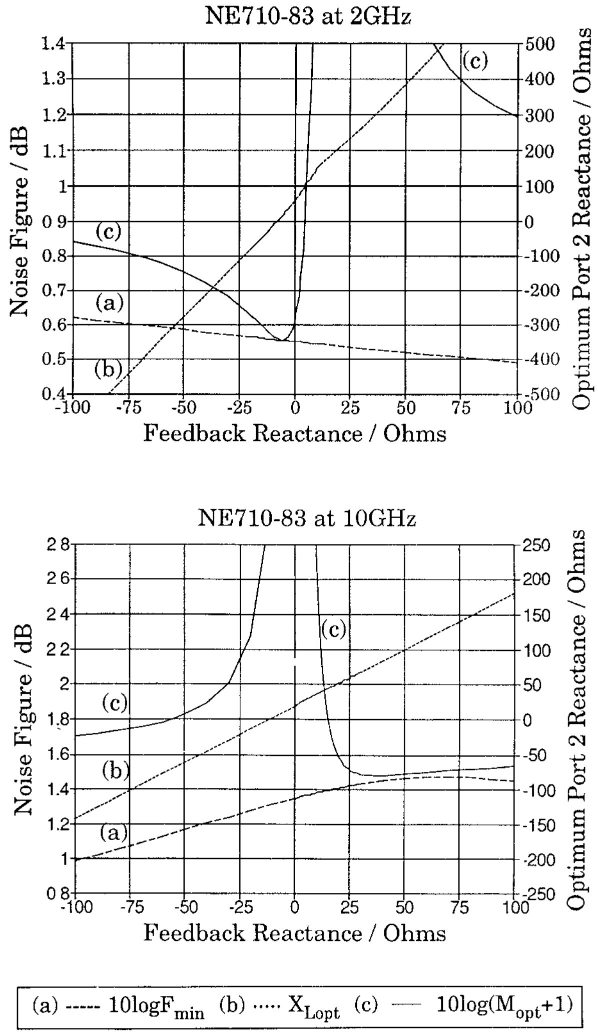

Fig. 5. Computed optimum noise measure design parameters.

It will be seen from the plots of Fig. 5 that at 2 GHz the condition $Z_{in} = -Z_{opt}$ is approached, and the optimum noise measure in reflection mode comes very close to $(F_{min} - 1)$ for one value of feedback reactance. At 10 GHz the approach is less close. This difference arises from the lower available gain at the higher frequency in a transmission mode amplifier, and the invariance of optimum noise measure with lossless embedding [10]. Poole and Paul [11] have given equations for the optimum noise measure of two port amplifiers. Using these it can be shown that there is exact numerical agreement between the minimum value of M_{opt} for the reflection mode device, determined graphically from Fig. 5, and the optimum transmission mode noise measure for the same device at the same frequency. At 2 GHz, where the available gain in transmission mode is high, M_{opt} is very close to $F_{min} - 1$. At higher frequencies, as the transmission mode gain becomes lower, M_{opt} becomes noticeably higher than $F_{min} - 1$.

The extremum in the plot of $10\log_{10}(M_{opt} + 1)$ extends over the range of feedback reactance, near to zero, for which the device is unconditionally stable.

B. Gain and Bandwidth Design

When the feedback and load reactances required for optimum noise measure have been determined, the next stage in the design process is to derive an input transforming structure to generate the required gain. The power gain of the amplifier is given by the reflection coefficient of the negative resistance device

$$G = \left| \frac{Z_{in} - Z_s}{Z_{in} + Z_s} \right|^2 \tag{31}$$

where Z_s is the source impedance presented to the negative resistance device and Z_{in} is given by

$$Z_{in} = Z_{11} + jX_f - \frac{(Z_{12} + jX_f)(Z_{21} + jX_f)}{Z_{22} + jX_f + jX_L}. \tag{32}$$

To optimize the gain over a wide frequency band it would be necessary to adopt synthesis procedures related to those developed by Scanlan [6], Getsinger [7] and Aron [8]. However for narrow bandwidth designs it is sufficient to use (32) calculated at a spot frequency. A specified gain G can then be generated for example by first resonating the reactive part of Z_{in} and then designing a network to transform the impedance at port two of the circulator into R'_s, where

$$R'_s = \frac{1+\sqrt{G}}{1-\sqrt{G}} R_{in}$$

or

$$R'_s = \frac{1-\sqrt{G}}{1+\sqrt{G}} R_{in}.$$

The choice of the value for R'_s depends on stability considerations. While both values will give a finite gain G in (31), one of the two values will give a circuit with an exponentially increasing transient response, i.e., it will oscillate. In most cases the stable circuit results from the condition for which the total circuit resistance is positive, i.e., $R'_s + \mathrm{Re}(Z_{in}) > 0$.

III. Experimental Amplifier

An experimental 10 GHz negative resistance low noise FET amplifier was designed and built using the NE710-83 packaged 0.3 μm GaAs FET. As a starting point for the design at 10 GHz, the following values for the optimum feedback ($X_{f\,opt}$) and load ($X_{L\,opt}$) reactances, as determined from Fig. 5, were used:

$$X_{f\,opt} = 36.75\ \Omega$$
$$X_{L\,opt} = 78.50\ \Omega.$$

With these reactances the predicted noise measure and input impedance are

$$M_{opt} = 0.406$$
$$Z_{in} = -14.92 + j18.38\ \Omega.$$

For a nominal gain of 10 dB at 10 GHz the required source impedance is

$$Z'_s = 28.72 - j18.38\ \Omega.$$

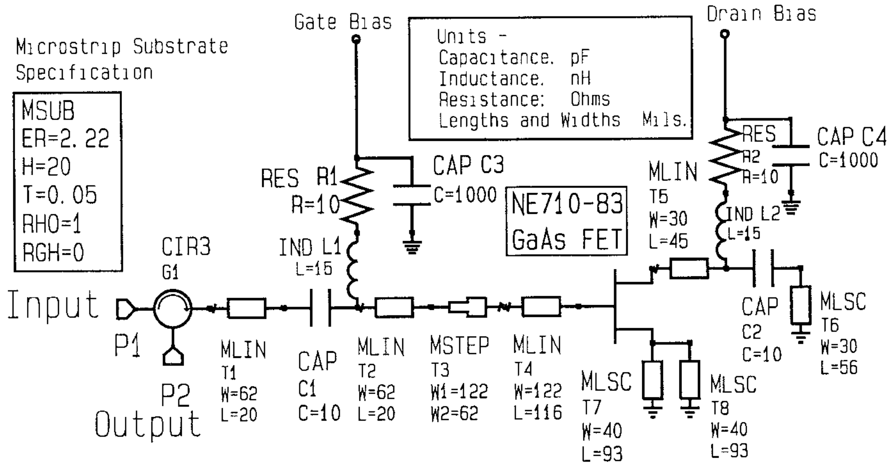

Fig. 6. Negative resistance low noise amplifier design schematic.

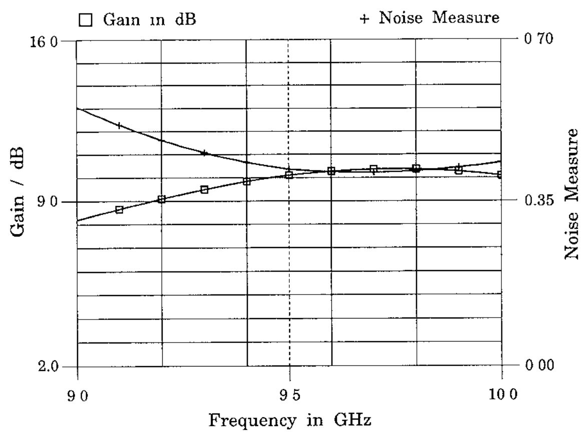

Fig. 7. Predicted gain and noise measure for experimental low noise amplifier.

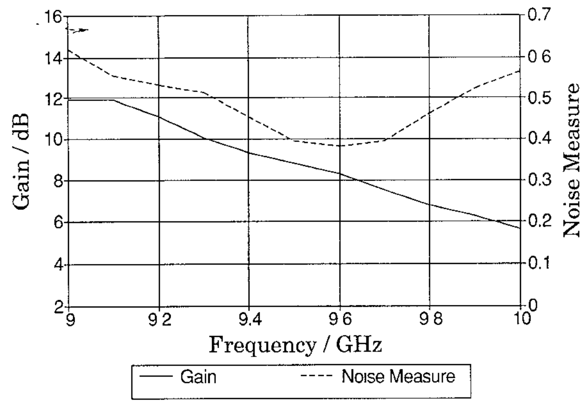

Fig. 8. Gain and noise measure for experimental GaAs FET reflection mode low noise amplifier.

The 50 Ω impedance on port two of the circulator can be transformed into this value of Z_s' using a transmission line of 25.34 Ω characteristic impedance and electrical length 30.4°. Short circuit transmission line stub parameters were calculated to generate the required reactances for the drain termination and the source feedback.

The circuit was then optimized for minimum noise measure and flat gain over the 9.5 to 10 GHz band, using the commercial microwave CAD package LIBRA. The final microstrip circuit schematic and the predicted gain and noise measure are shown in Figs. 6 and 7. The notation used for the circuit element parameters is that used by LIBRA [9].

The amplifier circuit was built on microstrip using dielectric material with dielectric constant 2.22. The measured gain and noise measure results are shown in Fig. 8, showing the noise measure close to the design figure, although the gain response peaks at a lower frequency than predicted. This discrepancy is attributable to the inadequate modeling of the dc blocking capacitors used in the gate and drain circuits, and to the effects of the deviation of the circulator impedance from 50 Ω.

IV. Conclusion

A theory has been derived for the design of an optimum noise measure negative resistance reflection amplifier using a two port device, with series reactive feedback and a reactive termination on port two.

It has been shown theoretically that the noise measure depends only on the values of these reactances and not on the parameters of the input transforming network, so long as it is lossless. The transforming network is designed to give the required gain and bandwidth characteristic whilst maintaining stability.

It is thus possible to obtain arbitrarily high gain, albeit with a limited gain bandwidth product, whilst maintaining optimum noise measure. This is a notable difference from the stable transmission mode amplifier case, where the gain has an upper limit when the input circuit is tuned for optimum noise measure. To achieve arbitrarily high gain and optimum noise measure in such an amplifier it would be necessary to add lossless positive feedback.

The optimum reactances have been calculated for a typical GaAs FET at two different frequencies, using the manufacturer's S-parameter and noise data. An experimental amplifier has been built and tested and shown to give noise measure results close to those predicted. The theoretical and practical results show that noise performance comparable to that of a conventional transmission mode amplifier using the same active device can be achieved.

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References

[1] H. Tohyama and H. Mizuno, "23 GHz band GaAs MESFET reflection type amplifier," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 408–411, May 1979.

[2] S. Nicotra, "13GHz FET negative resistance 0.5 Watt amplifier," in *Ninth European Microwave Conference Digest*, *Rec.* 1979 pp. 303–307.

[3] P. Penfield, Jr., "Noise in negative resistance amplifiers," *IRE Trans. Circuit Theory*, vol. CT-7, pp. 166–170, June 1960.

[5] K. Hartmann and M. Strutt, "Changes of four noise parameters due to general changes of linear two port circuits," *IEEE Trans. Electron Devices*, vol. ED-20, no. 10., Oct. 1973.

[6] J. O. Scanlan and J. T. Lim, "A design theory for optimum broadband reflection amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 504–511, Sept. 1964.

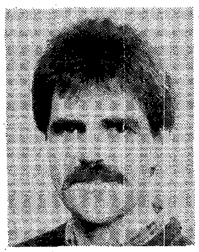
[7] W. J. Getsinger, "Prototypes for use in broadbanding reflection amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 486–497, Nov. 1963.

[8] R. Aron, "Gain bandwidth relations in negative resistance amplifiers," *Proc. IRE*, vol. 49, pp. 355–356, Jan. 1961.

[9] *EEsof Element Catalog*, EEsof Inc., 1989.

[10] H. A. Haus and R. B. Adler, *Circuit Theory of Linear Noisy Networks*. New York: Wiley, 1959.

[11] C. R. Poole and D. K. Paul, "Optimum noise measure terminations for microwave transistor amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1254–1257, Nov. 1985.



Peter Gardner was born in Welford, Northants, U.K. in 1958. He was awarded an open scholarship at Corpus Christi College, Oxford, in 1977, and received the B.A. degree in physics, first class, from the University of Oxford in 1980. He then studied solid state electronics at the University of Manchester Institute of Science & Technology (UMIST), U.K.

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